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Mathematical Science and Transient Simulation of 1-D Heat Exchanger

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Abstract - Transient simulation of heat exchanger is required to obtain the dynamic performance of the heat exchanger system. For prediction of the transient performance of heat exchanger a homogeneous model with vapor liquid two phase flow inside is used. To obtain the numerical solution of the model without solving a large set of non-linear equations simultaneously an efficient finite difference method is proposed. The method is capable of predicting the refrigerant temperature distribution, velocity of refrigerant, tube wall temperature as a function of position and time. A single tube heat exchanger with refrigerant R22 as working fluid was chosen as a sample and some tests were carried out to determine its transient response. The examination of results indicates that the theoretical model provides a reasonable prediction of dynamic response which is useful in designing a controllable compressor to reduce power consumption. Transient behavior of refrigerant has been obtained using MATLAB programs.

Index Terms— *Vapor liquid two phase flow, Homogeneous Model, set of non linear equations, finite difference method, and MATLAB program.*

1 Introduction

Analysis of heat exchanger with two phase flow begins with the most general principles governing the behavior of all matter, namely, conservation of mass, momentum and energy. These principles can be expressed mathematically at every point in space and time by local, instantaneous field equations. However the exact solution to these equations is almost impossible and very expensive, requiring the tracking of many convoluted liquid-vapor interfaces that change continuously in time. Instead, the usual procedure is to average the local, instantaneous equations in either time or space, or both. Although we lose information in the process, the resulting equations yield accurate solutions to a wide variety of practical problems so long as the averaged variables bear some resemblance to the actual situation, that is, so long as the flow is not too chaotic. During the averaging, the two phases may be treated together to obtain averaged variables for a two-phase mixture; alternatively, treating each phase separately, we obtain averaged variables for both phases. The better procedure yields the Homogeneous model, which is a bit more general and useful. A usual homogeneous model consists of three field equations: averaged mass, momentum and energy equations for the liquid and vapor phases. The model mainly takes account of refrigerant density, mass flow, and refrigerant temperature inside tube. The refrigerant mass flow rate, in general, is continuously changing, causing changes in refrigerant distribution in the system and because of two-phase flows inside the tube; the local heat transfer coefficient varies in a great range at different locations. This varying local heat transfer coefficient results in an uneven temperature distribution of the heat exchanger. In applications, to fully simulate the above process inside the tube demands a transient model of the system[i].

Therefore, homogeneous model of heat exchangers is governed by system of nonlinear partial differential equations i.e. the governing conservation equations (Navier-Stokes equations of CFD). A sub-class of numerical technique was found[ii] for solution of such model i.e. the phase-independent finite difference methods. In the finite difference approach, the governing conservation equations are approximated by a finite difference scheme that typically consists of dividing the heat exchangers into a number of elements, and each element is defined with its own state properties. The formulation for any element is phase-independent and therefore identical in all the phases.

2 Theoratical analysis

To predict this important phenomenon, the transient model should be fast enough to be practical. This requires the identification of suitable assumptions that can simplify the mathematical form without loss of relevant detail. Efficient numerical techniques are also necessary to reduce the computation time, thereby allowing the model to run as close to real-time as is possible, while also constraining errors to acceptable limits.

The common assumptions found to be made in the model are:

- Flow in the heat exchangers is one dimensional
- Axial conduction in the refrigerant is negligible
- Liquid and vapor refrigerant in the heat exchangers are in thermal equilibrium
- Effects of pressure wave dynamics are negligible
- Expansion is isenthalpic
- Compression is isentropic or polytrophic
- Thermal resistances of metallic elements in the system are negligible in comparison with their capacitances.

In addition to the above classifications, the flow of two-phase refrigerant can be modeled using a homogenization[iii]. In the homogenous model, the liquid and vapor phases are considered to be in thermal equilibrium and moving at the same velocity. This assumption is based on the belief that differences in the variables will promote momentum, energy, and mass transfer between the phases rapidly enough so that equilibrium is reached[iv].The governing equations for the Homogeneous model for vapor liquid two phase flow inside heat exchanger are as follows:

Refrigerant Side :-

Continuity Equation :-

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0 \ldots (1)$$

$$\rho = \alpha \rho_v + (1 - \alpha)\rho_l \ldots .. (2)$$

$$\rho_v = vapor\ density, \rho_l = liquid\ density$$

$$u = \frac{\dot{m}}{\rho}, \dot{m} = mass\ flow \ldots .. (3)$$

$$\alpha = \frac{1}{1 + \frac{\rho_v(1-z)}{\rho_l(z)}}, z\ is\ vapor\ quality \ldots .. (4)$$

Momentum Equation :-

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \beta \rho u^2}{\partial x} = -\frac{\partial P}{\partial x} \ldots \ldots (5)$$

$$\beta = z^2\left(1 + \frac{\rho_l(1-\alpha)}{\alpha \rho_v}\right) + (1 - z^2)\left(1 + \frac{\alpha \rho_v}{\rho_l(1-\alpha)}\right) \ldots \ldots (6)$$

Energy Equation :-

$$\frac{\partial \rho T}{\partial t} + \frac{\partial \rho u T}{\partial x} = \frac{A_i}{A_{tub}} U_i(T_w - T_f) \ldots \ldots (7)$$

Energy Equation for Wall Side :-

$$C_{pw} M \frac{\partial T_w}{\partial t} = \frac{U_o A_o}{C_{pa}}(h_a - h_w) - U_i A_i (T_w - T) \ldots .. (8)$$

To close the system of equations, constitutive equations heat transfer coefficients and thermal properties of the refrigerant and air are included. The single phase heat transfer coefficient is calculated by the Turaga et al. correlation[v]. Chen's correlation[vi] is adopted to estimate the local heat transfer coefficient of two-phase condensing flows. Thermal properties of the refrigerant (R-22) and air are calculated directly from the Refprop software.

Initial conditions

In this study, we are more interested in the dynamic response of the heat exchanger[vii] to variations of the system operation parameters; the heat exchanger is therefore assumed to be in the steady state or in equilibrium position initially. In the equilibrium state, the system should also obey the basic governing equations, and the only difference from the transient state is that all terms involved with time derivative in Equations (1), (5), (7) and (8) are set to zero. The solutions of the basic governing equations in the equilibrium state are used as the initial conditions of the system.

Boundary Conditions

The boundary conditions applied are the refrigerant conditions at the tube inlet and air conditions onto the heat exchanger coil:

mf / x = 0 = mf in
pf / x = 0 = pf in
α / x = 0 = α in
T / x = 0 = T in

3 NUMERICAL SOLUTION

To solve the above mathematical model numerically, the governing non linear equations are linearized by using quazi linearization and after that discredited using implicit finite difference scheme[viii] which results in algebraic equations as follows:

$$-\frac{r}{4}u_{j-1}^n \rho_{j-1}^{n+1} + \rho_j^{n+1} + \frac{r}{4}u_{j+1}^n \rho_{j+1}^{n+1} = \rho_j^n - \frac{r}{2}\left((\rho u)_{j+1}^n - (\rho u)_{j-1}^n\right) - \frac{r}{4}u_{j-1}^n \rho_{j-1}^n + \frac{r}{4}u_{j+1}^n \rho_{j+1}^n$$

$$-\frac{r}{4}u_{j-1}^n (\rho u)_{j-1}^{n+1} + (\rho u)_j^{n+1} + \frac{r}{4}u_{j+1}^n (\rho u)_{j+1}^{n+1}$$
$$= (\rho u)_j^n - \frac{r}{2}\left(\left((\beta \rho u^2)_{j+1}^n - (\beta \rho u^2)_{j-1}^n\right) - \left((P)_{j+1}^n - (P)_{j-1}^n\right) - \frac{r}{4}u_{j-1}^n (\rho u)_{j-1}^n\right.$$
$$+ \frac{r}{4}u_{j+1}^n (\rho u)_{j+1}^n$$

$$-\frac{r}{4}u_{j-1}^n (\rho T)_{j-1}^{n+1} + (\rho T)_j^{n+1} + \frac{r}{4}u_{j+1}^n (\rho T)_{j+1}^{n+1}$$
$$= (\rho T)_j^n - \frac{r}{2}\left((\rho u T)_{j+1}^n - (\rho u T)_{j-1}^n\right) - \frac{r}{4}u_{j-1}^n (\rho T)_{j-1}^n + \frac{r}{4}u_{j+1}^n (\rho T)_{j+1}^n$$
$$+ \frac{U_i A_i}{A_{tub}}\left(T_{w_j}^{\ n} - T_j^n\right)$$

$$C_{pw} M_w \frac{(T_w)_j^{n+1} - (T_w)_j^n}{\Delta t} = \frac{U_o A_o}{C_{pa}}(h_a - h_w) - U_i A_i (T_{w_j}^{\ n} - T_j^{\ n})$$

Where, r = dt/dx.

Above algebraic equations are in tridiagonal matrix form which it is solved by using LU decomposition in the present work.

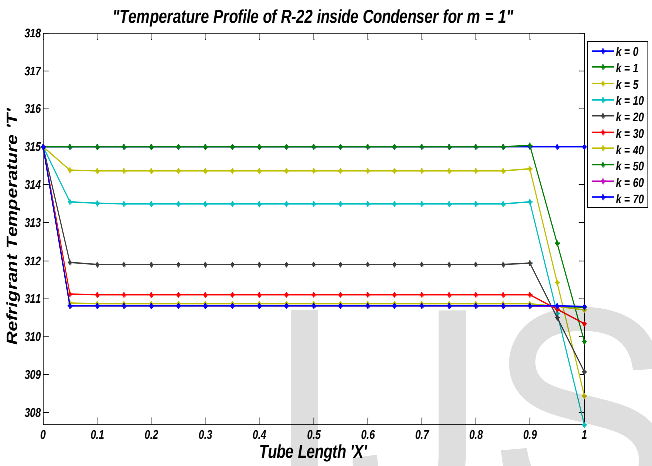
Heat Exchanger Simulation

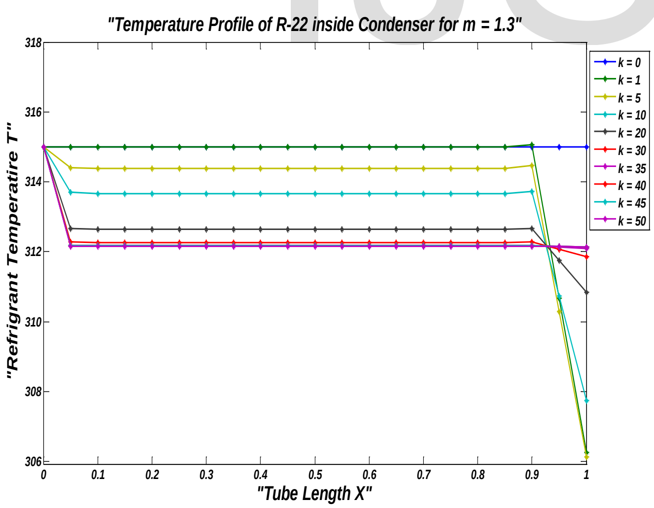
To simulate the problem the tube is considered of the unit length, inside diameter 0.0093 fetes and outside diameter 0.011 fetes. Now to start simulation, we apply the equilibrium state conditions together with the boundary conditions & heat transfer effects[ix]. First of all fluid enters in the tube in its superheated vapor form, and then by condensation it reduces temperature by releasing the heat. Fluid comes to its saturated vapor form at which it comes in two phase vapor liquid zone. Two phase zone is finished when vapor quality becomes very low; at that point fluid is in saturate liquid state. In the inverse problem, all fluid enters in its saturated liquid form, and then by evaporation it increase temperature by absorbing the heat and fluid comes in two phase liquid vapor zone. Two phase

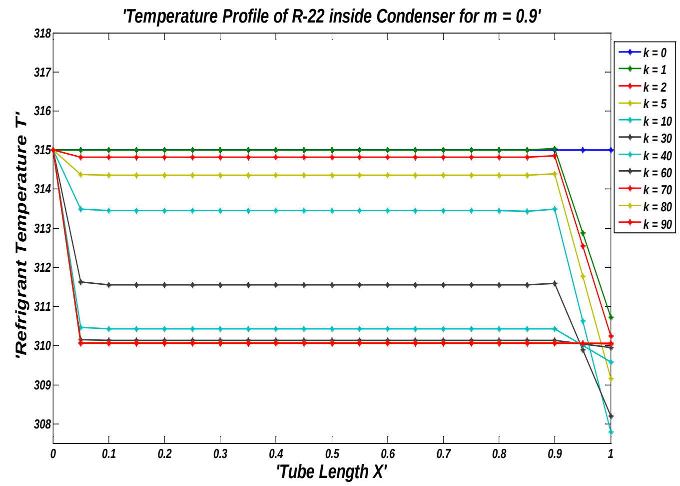
zone is finished when vapor quality becomes very high; at that point fluid is in superheated vapor state.
The above simulations are done by using programs based on MATLAB platform.

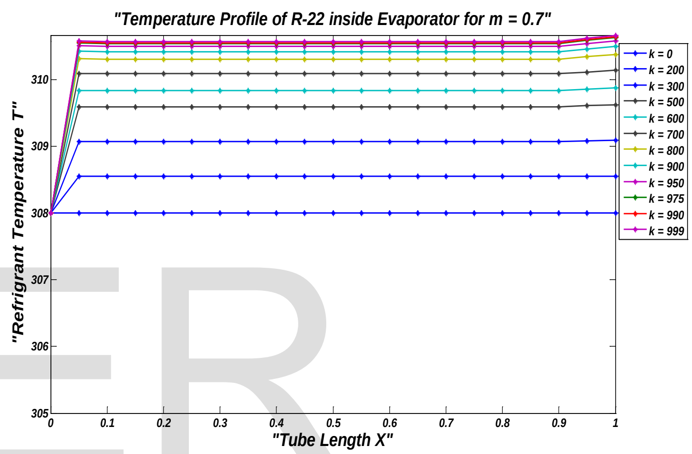
Result & Discussion

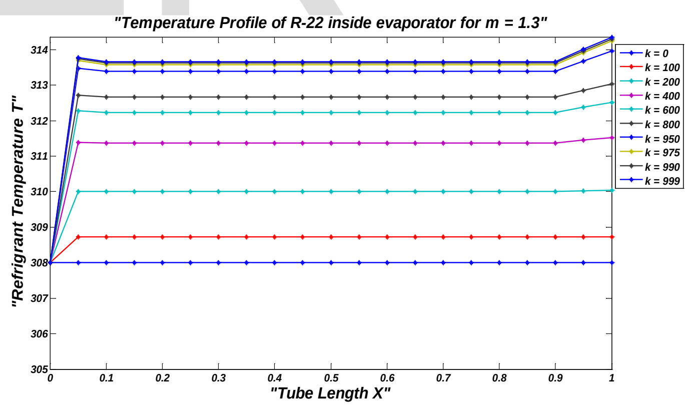
With the Matlab program for certain inputs graph of temperature inside tube on length scale is obtained for different time steps. These time steps are chosen to compute the difference between time steps of unsteady to steady behavior of heat exchanger. After obtaining the steadiness for the same time step graph of temperature inside tube is obtained by giving certain changes in mass flow inputs. Resulted Graphs are shown as follows:

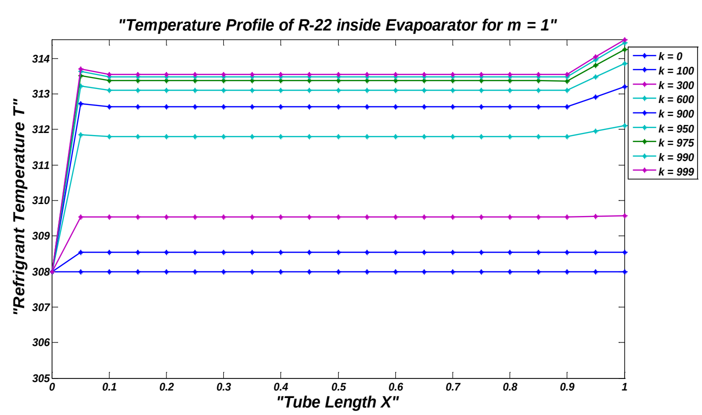












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4 Conclusion

Simulation of the homogeneous model to vapor-liquid flow for predicting the transient performance of a heat exchanger is presented. The simulation of the model is done by using efficient finite difference scheme which is capable of predicting distributions of the refrigerant temperature in both position and time domain without solving large number of nonlinear algebraic equations. The dynamic behavior of the heat exchanger is also investigated with a step increases and step decreases in the inlet refrigerant flow rate after obtaining steady state which results in variation of time steps and two phase length that is clearly obtained by using the present simulation. This knowledge of the dynamic characteristics of a heat exchanger is important to the design and control of many air-conditioning and refrigeration system.

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